New "Natural Neighbor" Meshless Method for Modeling Extreme Deformations and Failure

The objective of this work is to develop a fully Lagrangian approach based on a "natural neighbor" discretization technique to model extreme deformation and failure for analyses such as earth penetration and dam failure. In these problems, our standard Lagrangian finite-element approach fails, due to mesh tangling, and our Eulerian codes do not allow us to track particles and free surfaces to the degree necessary. Meshless particle methods, such as smooth particle hydrodynamics (SPH) and element-free Galerkin (EFG), have been used for modeling such large deformations but have a variety of numerical problems that the new natural neighbor method can potentially solve. To use the new approach successfully, issues such as numerical integration, time-step calculation, and adaptive point insertion are researched.

Project Goals

The goal of this work is to develop a more stable, more accurate meshless particle approach by overcoming the numerous problems inherent in these methods. The new approach will provide an improved method for modeling extreme events such as earth penetration and dam failure. Furthermore, because it is meshless, the approach can be used for applications where nondestructive characterizations are available, such as as-built weapons analysis and biomechanics. Overall, we will be able to solve a much larger class of problems.

Relevance to LLNL Mission

High-rate penetration dynamics has been identified as a challenge area in engineering and our new particle methods apply directly to applications in that field. Along with earth- and armor-penetration problems, vulnerability evaluation of infrastructures such as dams can be analyzed. As-built x-ray tomography of NIF targets and *in-vivo* MRI imaging for biomechanics create "point clouds" and are other good examples of where a meshless method could be exploited to expedite stress analyses.

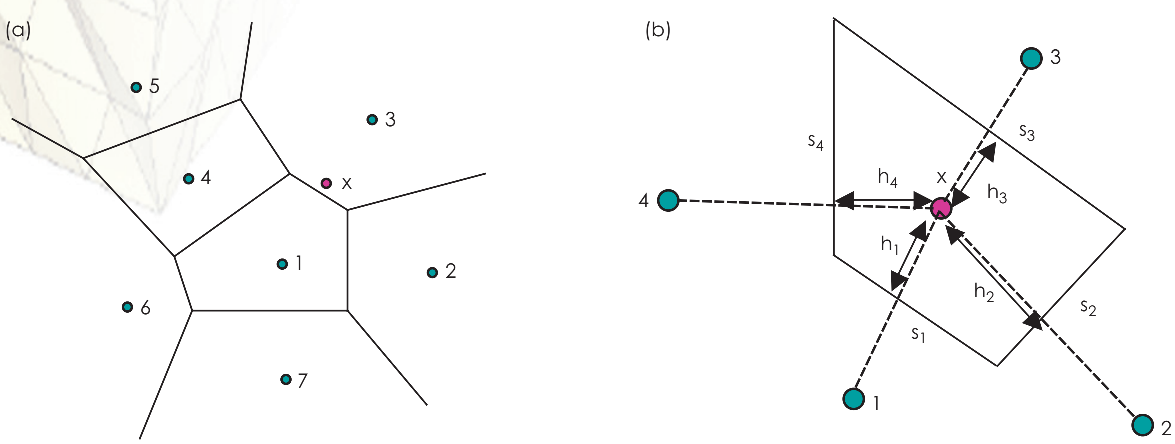

Figure 1. (a) Cloud of seven points and associated Voronoi diagram. (b) Considering the arbitrary location, **x**, in (a), a secondary local Voronoi cell is formed about this location **x** using its "nearest neighbors," points 1 to 4.

Engineering Modeling and Simulation

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FY2005 Accomplishments and Results

We implemented the natural neighbor interpolation method during FY2004. The Laplace shape function is defined from a Voronoi diagram of the cloud of points. Consider the 2-D cloud of points and the associated Voronoi diagram in Fig. 1(a). The shape function, N_i, for node $i = 1,4$ evaluated at the location, $\boldsymbol{x}$, is computed from the local Voronoi cell in Fig. 1(b), about the point, $\boldsymbol{x}$, using the lengths, s_i (the length of the edge), and h_i (the distance between $\boldsymbol{x}$ and p_i), in the equation,

$$N_i(x) = \frac{s_i / h_i}{\sum_{j=1}^{5} s_j / h_j}$$

We used a nodal integration scheme of the weak Galerkin form, where the strain at a node, i, is volume-averaged over the associated Voronoi cell using Green's theorem.

In 2005, we found that the method is, in fact, rank stable but is not stable in the H_1 norm. The problem was fixed using a perturbation stabilization technique that subdivides the Voronoi cell into subelements. This eliminated the weak mode present in the original implementation (Fig. 2). This technique was also successfully applied in FY2005 to the piecewise linear interpolation.

The Voronoi diagram provides integration domains for the new meshless method. Voronoi diagrams are simple to produce on convex domains but not so on nonconvex domains. Furthermore, Voronoi cells need to be capped at perceived boundaries. FY2005 work continued this effort. Figure 3 shows the resulting trimmed Voronoi diagram from a cloud of points using the "alpha approach." Finally, a technique for computing a stable explicit time step was developed by computing an upper bound on the maximum frequency of the discrete system. Until now, no method has been available for doing this for arbitrary meshless discretizations.

FY2006 Proposed Work

FY2006 will be dedicated to finishing NIKE3D/DYNA3D small deformation implementations, and extending the work to handle large deformations and damage. Tasks also include exploring different damage models and implementing adaptive point insertion.

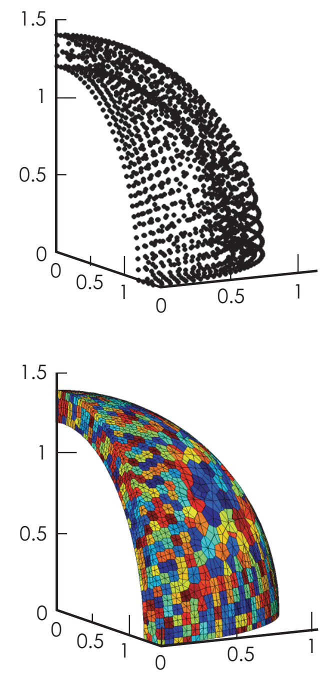

Figure 3. Cloud of points and trimmed Voronoi cells.

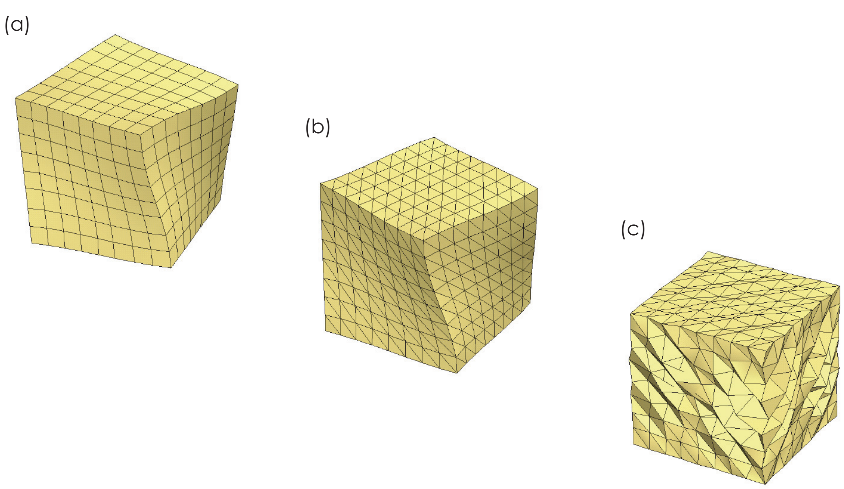

Figure 2. First eigenmode from (a) FEM 0.258 Hz; (b) NN (stabilized) 0.253 Hz; (c) NN (unstabilized) 0.161 Hz, illustrating the poor behavior of an unstabilized natural neighbor discretization.